LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

THIRD SEMESTER – **NOVEMBER 2014**

PH 3506 - MATHEMATICAL PHYSICS

Date : 31/10/2014   Dept. No.   Max. : 100 Marks
Time : 09:00-12:00

PART – A (10 x 2 = 20 Marks)

Answer ALL questions:

1. Compute the principal value of $\sqrt{i}$.
2. State Cauchy's integral theorem.
3. Calculate $grad\ \varphi$ if $\varphi = (x-2)(y+2)$ at $(1,2)$.
4. Evaluate $\int_0^2 \int_0^2 (x^2 + y^2)\, dx\, dy$
5. Determine the fundamental period of $\cos x$ and $\sin 2x$.
6. Using Laplace integral, evaluate $\int_0^{\infty} \frac{\cos \omega\ d\omega}{1+\omega^2}$
7. What is a normal matrix? Give an example.
8. Prove that the product of two unitary matrices is also unitary.
9. Using trapezoidal rule, evaluate $\int_0^2 y\, dx$ from the following data

x	0	0.5	1.0	1.5	2.0
y	1.000	0.800	0.500	0.308	0.200

10. Given $\frac{dy}{dx} = -y$ with $y = 1$ at $x = 0$. Find $y(0.02)$ using Euler's method.

PART – B (4 x 7.5 = 30 Marks)

Answer any FOUR questions:

11. (i) Evaluate $\int_i^{-1} z\, e^{\left(\frac{z^2}{2}\right)}\, dz$

    (ii) Integrate $\oint \frac{dz}{z^2+1}$ counter clockwise around a circle with $|z - i| = 1$.

12. If $\vec{u} = yz\,\vec{i} + zx\,\vec{j} + xy\,\vec{k}$ and $f = xyz$, find $curl\ (f\vec{u})$.

13. Find the Fourier series of the function $f(x) = \begin{cases} -1, & if\ -2 < x < 0 \\ 1, & if\ \ \ 0 < x < 2 \end{cases}$

14. Diagonalize the matrix $\begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$

15. Fit a straight line by least squares method for the following data

x	0	5	10	15	20
y	7	11	16	20	26

16. State and prove Green's theorem in the plane.

PART – C (4 x 12.5 = 50 Marks)

Answer any FOUR questions:

17. (i) Find the real and imaginary parts of $f(z) = 2iz + 6\bar{z}$ at $z = 0.5 + 4i$.

(ii) Show that $\cos h^2 z + \sin h^2 z = \cos h(2z)$

(iii) Using Cauchy's integral, evaluate $\oint \dfrac{e^z \cos z}{\left(z - \frac{\pi}{2}\right)^2} dz$ counterclockwise around the circle with $|z| = 2$.

18. (i) Find the directional derivative of $g = (x^2 + y^2 + z^2)^{-1/2}$ at $(4, 2, -4)$ in the direction of $(1, 2, -2)$.

(ii) Using Gauss-divergence theorem, evaluate $\int_S (x^3\, dydz + y^3\, dzdx + z^3 dxdy)$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 4$.

19. Find the even and odd periodic half range expansions of the function

$$f(x) = \begin{cases} \frac{2}{L}x & if\ 0 < x < \frac{L}{2} \\ \frac{2}{L}(L - x) & if\ \frac{L}{2} < x < L \end{cases}$$

20. Determine the eigen values of $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 0 & -2 \\ -2 & -2 & 1 \end{bmatrix}$ and show that matrix A satisfies its own characteristic equation.

21. (i) Find the Lagrange interpolating polynomial of degree 3 for the following data

x	0	1	4	5
y	8	11	78	123

(ii) Compute the value of $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ for $x = 1.05$ from the following table

	1	1.05	1.10	1.15	1.20
	1	1.0247	1.0488	1.0724	1.0955

22. (i) Determine the value of 'a' for the function $u = e^{3x} \cos ay$ is harmonic and also find its harmonic conjugate.

(ii) Find the inverse of the linear transformation
$$x = -2a - 2b + 7c$$
$$y = 4a + 3b - 12c$$
$$z = -a + 2c$$

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